Multidisciplinary optimization based on Kriging surrogate models

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1 Introduction

Objective of this study is to present a new multidisciplinary optimization (MDO) approach based on Kriging surrogate models. The method proposed focuses on multidisciplinary analysis (MDA) involving various disciplinary solvers with feedback coupling (*i.e.* the MDA is modeled as a non linear system of equations). Moreover, it is assumed that gradients of the disciplinary solvers with respect to the design variables are not provided by the solvers and that their computation by finite differences is numerically intractable.

In this context, surrogate based optimization approaches seem to be relevant and have shown efficiency in the context of optimization of black-box function (see [1]). However, to the best of our knowledge, there are only few attempts to adapt them to the MDO context. The proposed approach can be seen as a new MDO formulation in which each disciplinary solver is surrogate by a Kriging metamodel. Then, these surrogate models are adaptively enriched in promising areas of the design space with respect to the minimum value of the objective function. Obviously the difficulties lie in the research of the *promising areas* of the design space, given that the various disciplines are coupled, thus rendering non-trivial the choice of the most relevant input values. In order to properly define these areas, the probabilistic framework is retained. Hence, each disciplinary solver is surrogate by a Gaussian Process (Kriging) over the design and coupling variable spaces. Then, by carefully monitoring the uncertainty introduced by the use of these surrogate models in the MDO process, the proposed algorithm finds relevant points of the design space where the disciplinary surrogate models should be enriched.

2 Overview

Difficulties to set up the proposed strategy have been split into three main steps:

- Uncertainty propagation in a coupled system. Indeed, as the disciplinary Gaussian Process surrogate models are coupled through a non linear system (MDA), the resulting uncertainty on the objective function is not Gaussian and will be numerically evaluated. In practice the proposed strategy rely on polynomial chaos expansion PCE (see [2], [3]).
- As a consequence the objective function is modeled as non gaussian, second order random field over the design space. The discretization of this random field is achieved by an original approach proposed in [4]. The idea beside this original approach is to construct a continuous representation of the random field based on an interpolation of the discrete spectral representation (obtained by PCE at various points of the design space).
- Finally, an Expected Improvement EI criterion is derived in order to find relevant areas of the design space where the disciplinary surrogate models must be enriched. This criteria is optimized at each iteration of the proposed method. Then, the disciplinary surrogate models are improved (by adding the point that optimizes the EI to the design of experiments used to construct the disciplinary surrogate models).

3 First results and conclusions

The proposed approach has been tested on an academic case proposed in [5] involving two disciplines, two coupling variables and 3 design variables. The initial design of experiments for the construction of the disciplinary Gaussian Process surrogate models counts 5 points. Then, the proposed approach reaches convergence after 7 iterations. Finally, each disciplinary solver is called only 12 times. As the proposed approach is based on a probabilistic framework the minimum value of the objective function is modeled as a random variable which uncertainty is reduced during the iterative process. Figure 1 illustrates this uncertainty reduction by showing an histogram of the minimum value of the objective function at the first iteration and after the last iteration of the proposed method. It should be noted that the reference value for the minimum value on this example is closed to 3.1834, thus as presented by the right of Fig: 1 the proposed approach perfectly matches this result. Indeed, the mean value of the random variable modeling the minimum value of the objective function is approximately equal to 3.1819 and its coefficient of variation is around0.2%.

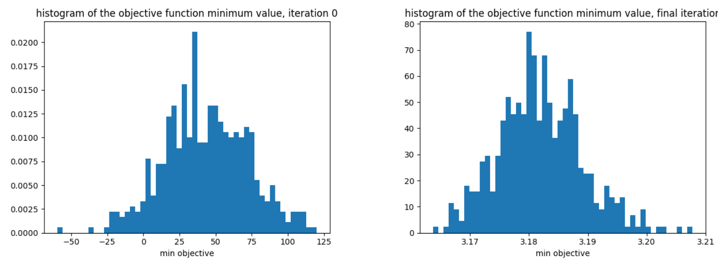

Fig. 1. Left: Histogram of the objective function minimum value at the initial step. Right: Histogram of the minimum value of the objective function after convergence of the proposed method.

In conclusion a new MDO formulation based on disciplinary surrogate models is proposed. Advantages of the proposed formulation can be summarized as,

- Each disciplinary surrogate model is built independently.
- In terms of numerical efficiency the MDA is never solved using the disciplinary solvers. This step is replaced by an uncertainty quantification process involving only the surrogate models.
- The number of calls to the disciplinary solver is reduced by carefully selecting the discretization points used to build the surrogate models. The surrogate models are thus only accurate in the area of the design and coupling variables space where the minimum is likely to be.
- As the probabilistic framework is used, the final results (minimum value and position) are random variables, which allow to easily quantifying the confidence on these results.

However, some remaining challenges are still open questions such as the number of coupling variables involved in the MDO problem. Indeed, a large number of coupling variables leads to large stochastic dimension in the uncertainty propagation steps which might be difficult to solve with our current implementation of PCE.

References

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